Existential graphs – a graphical logic of Ch. S. Peirce

Diagrammatic reasoning is the only really fertile reasoning. If logicians would only embrace this method, we should no longer see attempts to base their science on the fragile foundations of metaphysics or a psychology not based on logical theory; and there would soon be such an advance in logic that every science would feel the benefit of it.

Ch. S. Peirce, Prolegomena to an Apology For Pragmaticism, 1906

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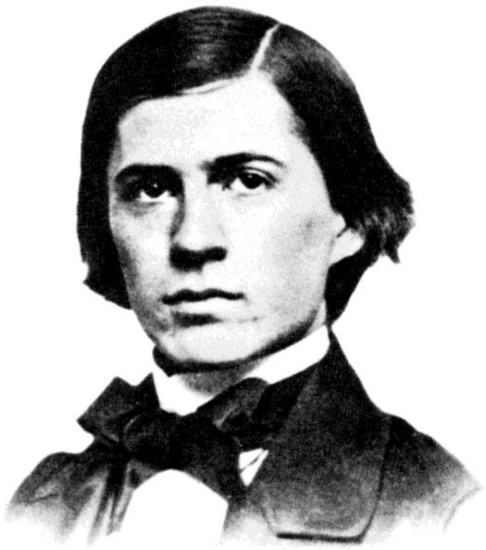
Charles Sanders Peirce

- American scientist, mathematician, logician and philosopher (1839 – 1914)
  - The son of Benjamin Peirce, a professor of mathematics at Harvard
- Interests:
  - Chemistry
  - Geodesics
  - Scientific methodology
  - Semiotics (study of signs)
  - Logic in broad and narrow sense
    - How do we acquire new knowledge?



Charles Sanders Peirce

- Founded a philosophical movement of pragmatism
  - Our understanding of an object or a concept reduces to practical effects of that object/concept
- Introduced the notion of abduction
- Developed a scientific method
  - Abduction – formulate a hypothesis
  - Deduction – determine its consequences
  - Induction – verify the consequences
- Founded semiotics (study of signs)
  - Icons, indices and symbols

Charles Sanders Peirce

- Logic in a narrow sense:
  - Developed syntax of more than a dozen of systems of logic
  - Introduced implication operation, Peirce's arrow and Sheffer stroke (40 years before Sheffer)
  - Introduced variables and quantifier symbols (∑ and ∏)
  - Developed a logic of relations (essentially predicates) two years before Frege (1870)
  - Developed complete syntax of quantified expressions (1883), almost identical to Russell & Whitehead (1910)
  - Developed two systems of graphical logic – entitative graphs и existential graphs

Existential graphs

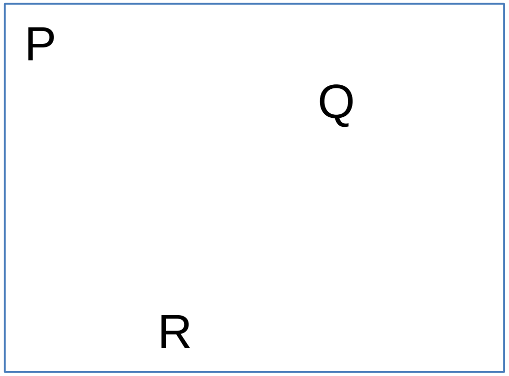
- Motivation:
  - Disappointment in symbolic logic:
    P^Q^R, R^Q^P, {P, Q, R} – three different situations
  - Impression of Venn diagrams (Euler circles) and an attempt to create a similar thing for the logic of relations
  - Attempt to make logical representation clear and iconic – intuitively reflecting its meaning
- Peirce considered existential graphs the pinnacle of his work and believed that they were the future

Existential graphs

- Three systems:
  - Alpha – equivalent to propositional logic
  - Beta – equivalent to first-order predicate logic
  - Gamma – contains elements of higher-order and modal logic (unfinished)
- Existential graphs were rediscovered in the middle of the 20th century
  - Their equivalence to the systems of classical logic was proved

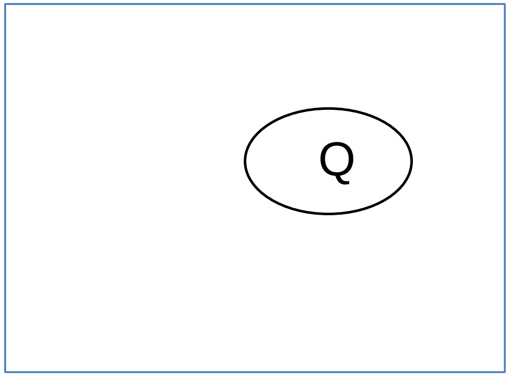
Existential graphs (alpha)

- *Atomic statements* are placed freely on the *sheet of assertions*
- Corresponds to (generalized) conjunction
  - Order does not matter
- Can be interpreted in PL as:
  - Three statements P, Q и R
  - One statement P^Q^R (or Q^R^P, or R^Q^P, etc.)
  - Two statements P and Q^R (or Q and P^R, or R and Q^P, etc.)



Existential graphs (alpha)

- *Circle* or *oval* around a statement denotes that the statement if false, i.e. its negation
  - Symbolizes a *cut* from the sheet of assertions
- Entire inventory:
  - Sheet of assertions
  - Atomic statements
  - Conjunction – juxtaposition
  - Negation – a cut

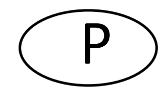


- This is enough to express any statement of propositional logic

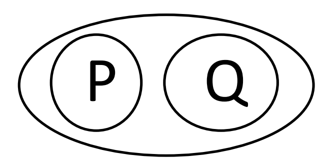
Symbolization in graphs alpha

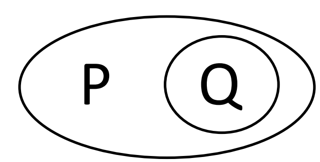
- Translation from PL to graphs:

P ⟶ P

$\neg P$ ⟶ 

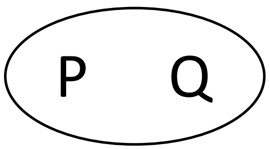
$P \wedge Q$ ⟶ P Q

$P \vee Q$ ⟶ 

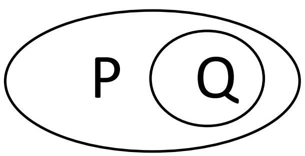
$P \rightarrow Q$ ⟶ 

Reading of graphs alpha

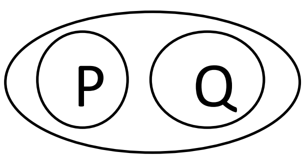
- Translation from graphs to PL:



$\longrightarrow$ $\neg(P \wedge Q)$ $\neg P \vee \neg Q$ $P \rightarrow \neg Q$ $Q \rightarrow \neg P$



$\longrightarrow$ $\neg(P \wedge \neg Q)$ $\neg P \vee Q$ $P \rightarrow Q$ $\neg Q \rightarrow \neg P$



$\longrightarrow$ $\neg(\neg P \wedge \neg Q)$ $P \vee Q$ $\neg P \rightarrow Q$ $\neg Q \rightarrow P$

Inference rules

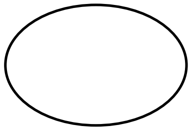
- 2 rules of equivalence:
  - A double cut can be drawn or removed around any subgraph
  - Any subgraph can be copied on the same level or a nested level. And vice versa – an existing copy on the same or a nested level can be removed
- 2 rules of inference:
  - On any *even* nested level any subgraph can be removed (generalization)
  - On any *odd* nested level any subgraph can be added (specialization)

Related notions

- Subgraph:
  - Any part of the graph where all cuts preserve all their content
- Double cut:
  - Two cuts nested in one another having nothing in between
- Subgraph nesting level:
  - Number of cuts around it
- Nested level:
  - Level to which you can "pass" without going outside of cuts

Special graphs

- Empty graph:
  - The most general truth – *True* constant
  - The only axiom of existential graphs
- Empty cut:
  - *False* constant, symbolizes a contradiction



Proofs

- Dynamic transformation of graphs – a movie
- To show the validity of the argument ($P \to Q$):
  - Transform the premises graph into the conclusion graph
- To show the equivalence of two graphs:
  - Transform each graph into the other
- To show the inconsistency of a graph:
  - Transform the graph to an empty cut
- To show that a graph is a tautology:
  - Transform an empty graph to that graph

Efficiency

- In traditional systems of logic:
  - More inference rules $\to$ shorter proofs
  - Less inference rules $\to$ longer proofs
- In existential graphs:
  - Only 4 inference rules
  - Proofs are usually shorter then in standard logic
- Unified representation of equivalent formulae:
  - One graph can be read in different ways
- Doing proofs in EG is probably easier then in PL

Existential graphs (beta)

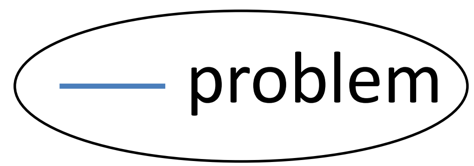
- *Lines of identity*:
  - Denote objects
  - Correspond to the existential quantifier
- *Predicates*:
  - Denote properties and relations

- *A cat is on a mat* ∃x, y (cat(x) ^ mat(y) ^ on(x, y))

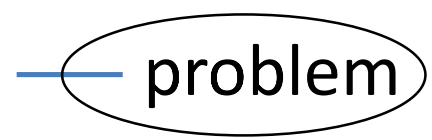
Existential graphs (beta)

- There is a problem

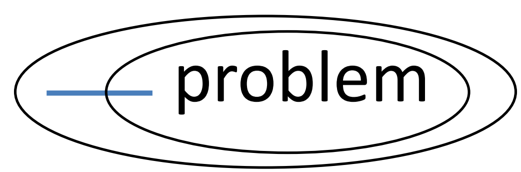
- There is no problem



- There is a non-problem

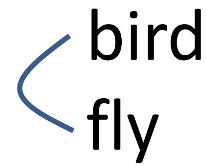


- There is no non-problem
(Everything is a problem)

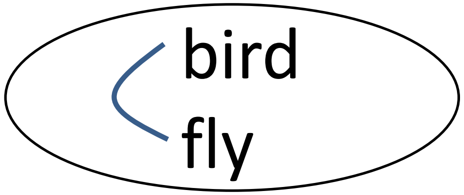


Existential graphs (beta)

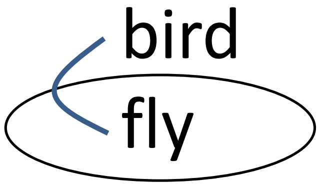
- Some birds fly



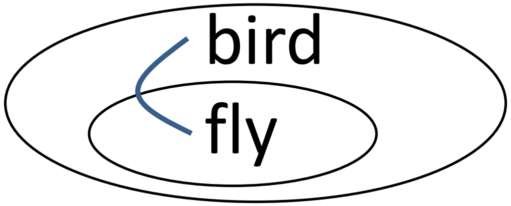
- No birds fly
  (If bird then does not fly)



- Some birds do not fly

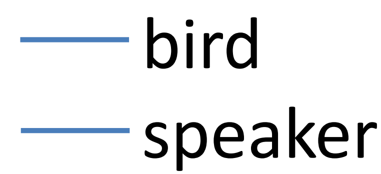


- All birds fly
  (If bird then flies)

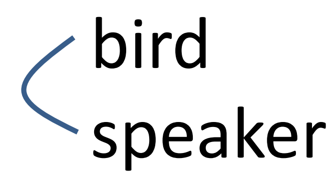


Identity and non-identity

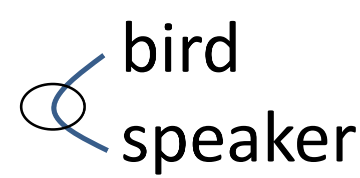
- There is a bird and a speaker
  (may be same or different)



- There is a speaker bird
  (the same object)

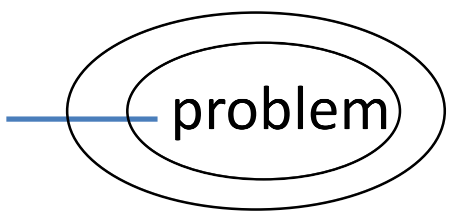


- There is a bird and separately –
  there is a speaker
  (different objects)



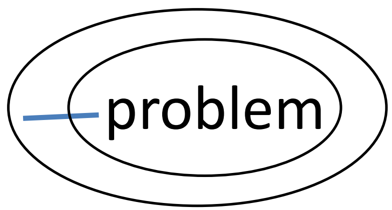
Inference rules (beta)

- The same 4 inference rules
  - But we have to allow them to interact with lines of identity
- Double cut
  - Ignore lines going through both cuts
  - But not the ones beginning/ending in between



Double cut:

There is a non-non-problem

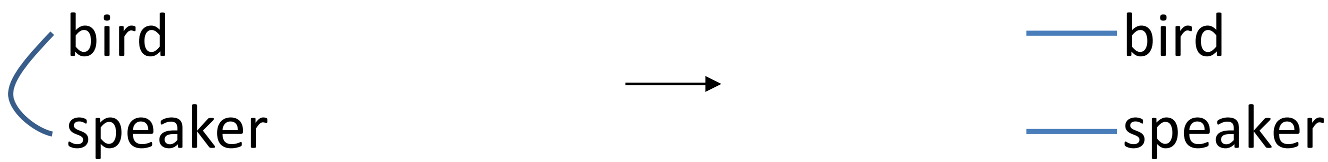


NOT a double cut:

Everything is a problem

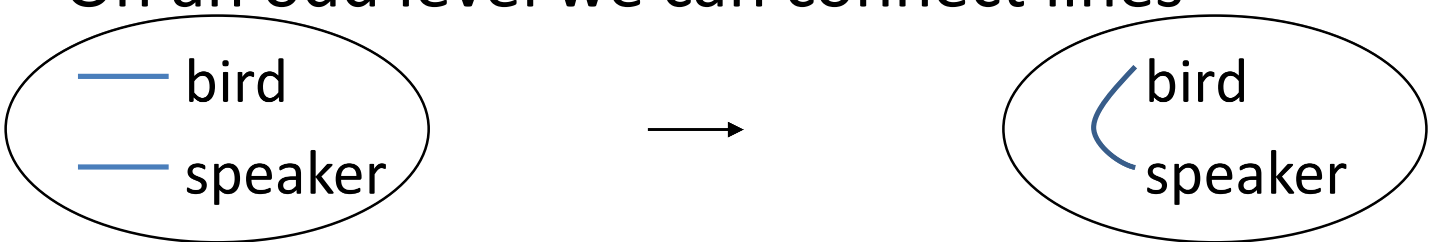
Inference rules (beta)

- On an even level we can break lines



There is a speaker bird — There is speaker and a bird

- On an odd level we can connect lines

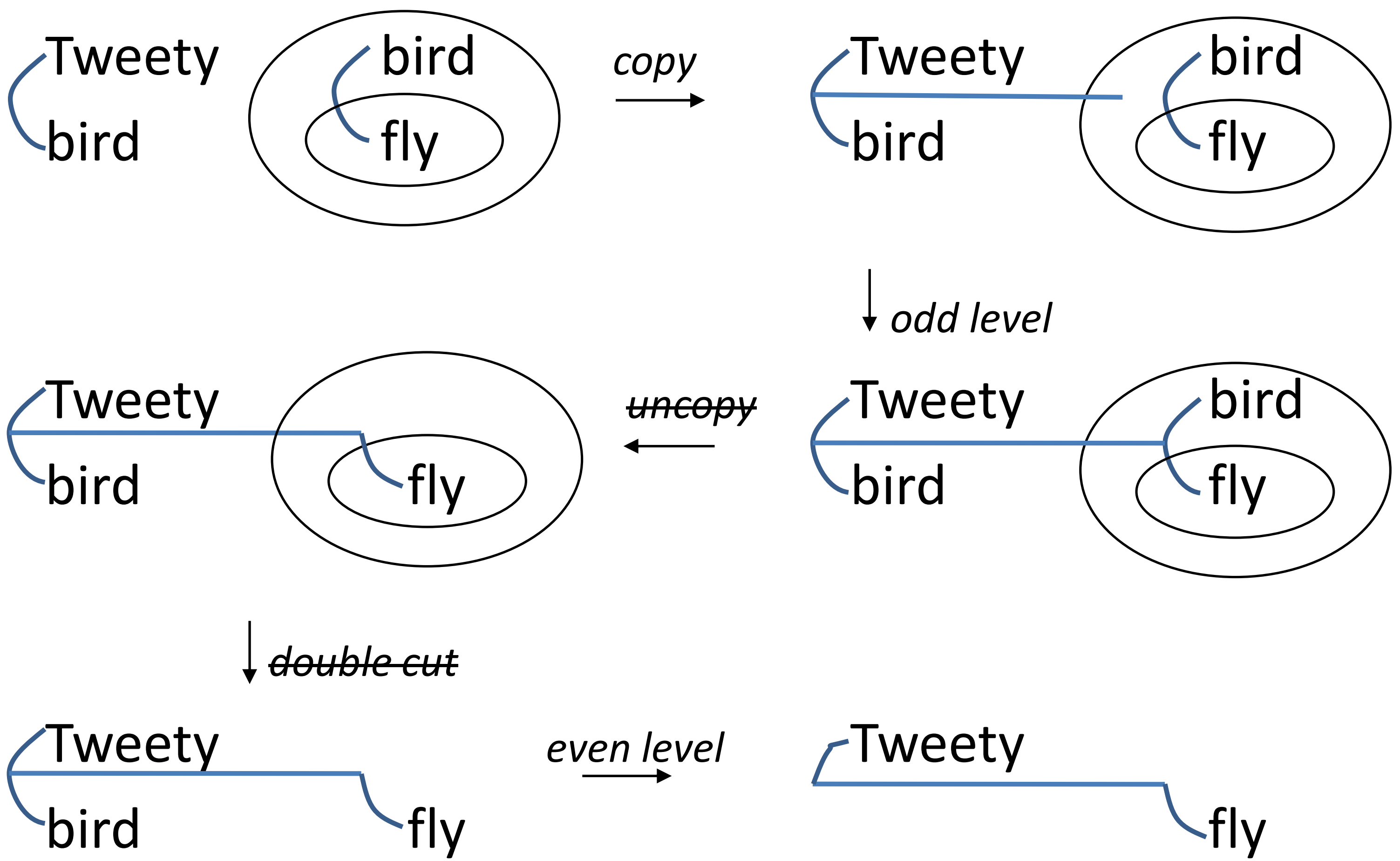


Either there is no bird or no speaker — There is no speaker bird

Inference rules (beta)

- Copying:
  - Lines can be extended and branched on the same level or into nested levels
  - A loose end of a line can be erased and removed from a nested level
  - A copy of a predicate can be connected by lines with the original predicate
  - A loop can be broken on the deepest nested level

Inference example



Thanks for your attention!
Questions?

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